JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 14, No. 4, October–December 2000

# Robust Heat Transfer Correlation for Turbulent Gas–Liquid Flow in Vertical Pipes

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**In this study a robust two-phase nonboiling heat transfer correlation for turbulent flow ($Re_{SL}$ >4000) in vertical tubes with different fluid flow patterns and fluid combinations was developed using experimental data available from the literature. The correlation presented herein originates from a careful analysis of the major nondimensional parameters affecting two-phase heat transfer. This model takes into account the appropriate contributions of both the liquid and gas phases, using the respective cross-sectional areas occupied by the two phases. A total of 255 data points from three available studies (which included the four sets of data) were used to determine the curve-fitted constants in the improved correlation. The performance of the new correlation was compared with two-phase correlations from the literature, which were developed for specific fluid combinations.**

## Nomenclature

| | | |
|---|---|---|
| $A$ | = | cross-sectional area, m$^2$ |
| $c$ | = | specific heat at constant pressure, kJ/(kg · K) |
| $D$ | = | inside diameter of the tube, m |
| $h$ | = | heat transfer coefficient, W/(m$^2$ · K) |
| $k$ | = | thermal conductivity, W/(m$^2$ · K) |
| $\dot{m}$ | = | mass-flow rate, kg/s |
| $Pr$ | = | Prandtl number, $c\mu/k$ |
| $Q$ | = | volumetric flow rate, m$^3$/s |
| $Re$ | = | Reynolds number, $\rho V D/\mu_B$ |
| $Re_{SL}$ | = | superficial Reynolds number, $\rho V_{SL} D/(\mu_B)_L$ |
| $V$ | = | average velocity in the test section, m/s |
| $V_{SG}$ | = | superficial gas velocity, $Q_G/(A_G + A_L)$, m/s |
| $V_{SL}$ | = | superficial liquid velocity, $Q_L/(A_G + A_L)$, m/s |
| $x$ | = | flow quality, $\dot{m}_G/(\dot{m}_G + \dot{m}_L)$ |
| $\alpha$ | = | void fraction, $A_G/(A_G + A_L)$ |
| $\mu$ | = | dynamic viscosity, Pa · s |
| $\rho$ | = | density, kg/m$^3$ |

*Subscripts*

| | | |
|---|---|---|
| $B$ | = | bulk |
| CAL | = | calculated |
| EXP | = | experimental |
| $G$ | = | gas |
| $L$ | = | liquid |
| SG | = | superficial gas |
| SL | = | superficial liquid |
| TP | = | two-phase |
| $W$ | = | wall |

## Introduction

IN many industrial applications, such as the flow of natural gas and oil in flowlines and wellbores, the knowledge of nonboiling two-phase, two-component (liquid and permanent gas) heat transfer is required. Numerous heat transfer coefficient correlations and experimental data for forced convective heat transfer during gas–liquid two-phase flow in vertical and horizontal pipes have been published over the past 40 years.[1] These correlations for two-phase flow convective heat transfer were developed based on limited experimental data and are only applicable to certain flow patterns.

Previously Kim et al.[1,2] identified 20 two-phase flow heat transfer correlations from previously published studies, and these correlations were compared with a large set of two-phase flow experimental data, for vertical and horizontal tubes including different flow patterns, and fluid combinations. Based on the tabulated and graphical results of the comparisons between those 20 correlations and the large set of experimental data available, appropriate correlations for different fluid combinations, flow patterns, and tube orientations were recommended by Kim et al.[1] Table 1 presents Kim et al.'s[1] recommended turbulent ($Re_{SL} > 4000$) two-phase heat transfer correlations for the four fluid combinations (water–air, silicone–air, water–helium, and water–freon 12) in vertical pipes. There were only three experimental studies found in the given literature, which included the measured parameters required to evaluate and develop correlations. They were the studies of Aggour,[3] Vijay,[4] and Rezkallah[5] (which included the four sets of data). The parametric range of variation for the 255 data points used in Table 1 can be found in Ref. 1.

From Table 1 the recommendation is made that for the water–air data of Vijay[4] the correlation of Shah[6] should be used for bubbly, slug, froth, bubbly-froth, and froth-annular flow patterns; for the silicone–air data of Rezkallah,[5] the correlation of Rezkallah and Sims[7] should be used for bubbly and bubbly-froth patterns; for the water–helium data of Aggour,[3] the correlation of Knott et al.[8] should be used for all of the flow patterns in Table 1 except the bubbly-slug and annular-mist flow patterns; and for the water–freon 12 data of Aggour,[3] the correlation of Aggour[3] should be used for all of the flow patterns listed in Table 1. Comparing the performance of the predictions for water–air and silicone–air data with the water–helium and water–freon 12 data in Table 1, it was more difficult to find a good correlation for water–air and silicone–air data, which could be applicable to several different flow patterns. Flow pattern identification for the experimental data was based on the procedures suggested by Govier and Aziz,[9] Griffith and Wallis,[10] Hewitt and Hall-Taylor,[11] and visual observation where available.

Those 20 correlations identified by Kim et al.[1] and the correlations referenced in Table 1 have some of the following important parameters in common: $Re_{SL}$, $Pr_L$, $\mu_B/\mu_W$ and either void fraction ($\alpha$) or superficial velocity ratio ($1 + V_{SG}/V_{SL}$). Because there was no single correlation capable of predicting heat transfer rate with good accuracy for all fluid combinations in vertical pipes, there appears to be at least one parameter (ratio), which is related to fluid combinations, that is missing from those correlations. To improve the applicability of the prediction of heat transfer rate in vertical turbulent two-phase flows regardless of fluid combination and flow pattern, a new robust correlation has been developed, and the accuracy of this new correlation has been compared with those of the recommended fluid combination dependent correlations in Table 1.

Received 7 June 1999; revision received 10 January 2000; accepted for publication 14 January 2000. 

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**Table 1 Results of the predictions for available experimental data using the recommended correlations by Kim et al.[1a]**

| | No. of data points within ±30%/total no. of data points for experimental study[b] used correlation: | | | |
|---|---|---|---|---|
| Flow pattern | Vijay[4]/Shah[6] | Rezkallah[5]/Rezkallah and Sims[7] | Aggour[3]/Knott et al.[8] | Aggour[3] |
| Bubbly | 25/25 | 20/20 | 9/10 | 6/6 |
| Slug | 12/17 | —— | 10/10 | 6/6 |
| Froth | 25/25 | 10/18 | 12/12 | 10/10 |
| Annular | 3/21 | 0/2 | 8/9 | 14/14 |
| Bubbly-slug | —— | —— | 1/2 | 3/4 |
| Bubbly-froth | 7/7 | 10/10 | 1/1 | 1/1 |
| Slug-annular | 1/2 | —— | 4/4 | 3/3 |
| Froth-annular | 4/4 | 0/6 | —— | —— |
| Annular-mist | 0/4 | —— | 0/2 | —— |
| All flow patterns | 77/105 | 40/56 | 45/50 | 43/44 |
| Mean dev., % | 21.42 | −17.98 | 3.85 | −1.0 |
| rms dev., % | 26.32 | 31.65 | 18.04 | 14.35 |
| Dev. range, % | −1.20 and 61.47 | −84.17 and 8.60 | −75.97 and 33.35 | −28.64 and 36.81 |

[a]Note: Blanks indicate no data for flow pattern. For studies, Vijay, water–air (105 data points); Rezkallah, silicone–air (56 data points); Aggour, water–helium (50 data points) and water–freon 12 (44 data points).
[b]$Re_{SL} > 4000$ only.

## Development of the New Correlation

Of the two-phase heat transfer correlations that have been published over the past 40 years, the vast majority were developed from limited experimental data and are only applicable to certain flow patterns and fluid combinations. Whereas most of these correlations were derived empirically based on a small set of experimental data, others were based on such concepts as the liquid acceleration model, the pressure drop model, and the separated flow model. Kim et al.[2] have presented descriptions of these concepts and identified the correlations that were developed based on each concept. A brief summary of their work related to the preferred correlations in Table 1 follows.

In the liquid acceleration model concept the assumption was made that the introduction of the gas phase into the two-phase heated test section acts only to accelerate the liquid phase and that the heat is transferred mainly by the liquid phase. Thus, the two-phase heat transfer mechanism could be considered as heat transfer to a single-phase liquid flow, with the liquid flowing at the actual mean (not the superficial) velocity in the heated test section. Therefore, the void fraction parameter explicitly appeared in the two-phase heat transfer correlations to estimate the in situ velocity of the liquid phase during gas–liquid two-phase flow. Two examples of this type of correlation are those of Aggour[3] and of Rezkallah and Sims.[7] This type of correlation was recommended[2] for extremely high-liquid Prandtl number (6300–7000), such as glycerin–air, and moderately high-liquid Prandtl number (61–77), such as silicone–air.

In the separated flow model concept, the assumption was made that the increased heat transfer in two-phase flow was attributed to the increase of the effective mixture velocity. The effective mixture velocity was defined as the sum of the single-phase liquid and gas superficial velocities. Therefore, the parameter $Re_{TP}$ or $(1 + V_{SG}/V_{SL})$ appeared in this type of correlation. Examples of these are the correlations of Knott et al.[8] and Shah,[6] and these correlations were recommended for the air–water and water–helium heat transfer experimental data, as can be seen from Table 1.

Because there is no single correlation capable of predicting turbulent heat transfer rate with good accuracy for all fluid combinations and different flow patterns in vertical pipes, there appears to be at least one parameter (ratio), which is related to fluid combination, that is missing from the earlier correlations. To improve the prediction of heat transfer rate in vertical turbulent two-phase flow, regardless of fluid combination and flow pattern, this study developed a new correlation described in the following section including Eqs. (1–12). This improved correlation uses a carefully derived heat transfer model that takes into account the appropriate contributions of both the liquid and gas phases, using the respective cross-sectional areas occupied by the two phases.

The void fraction $\alpha$ is defined as the ratio of the gas-flow cross-sectional area $A_G$ to the total cross-sectional area $A_G + A_L$,

$$\alpha = A_G/(A_G + A_L) \tag{1}$$

The actual gas velocity $V_G$ can be calculated from

$$V_G = \frac{Q_G}{A_G} = \frac{\dot{m}_G}{\rho_G A_G} = \frac{\dot{m}x}{\rho_G \alpha (A_G + A_L)} \tag{2}$$

Similarly, for the liquid $V_L$ is defined as

$$V_L = \frac{Q_L}{A_L} = \frac{\dot{m}_L}{\rho_L A_L} = \frac{\dot{m}(1-x)}{\rho_L (1-\alpha)(A_G + A_L)} \tag{3}$$

The total gas–liquid two-phase heat transfer is assumed to be the sum of the individual single-phase heat transfers of the gas and liquid, weighted by the volume of each phase present:

$$h_{TP} = (1-\alpha)h_L + \alpha h_G = (1-\alpha)h_L\{1 + [\alpha/(1-\alpha)](h_G/h_L)\} \tag{4}$$

There are several well-known single-phase heat transfer correlations in the literature. In this study the Sieder and Tate[12] equation was chosen as the fundamental single-phase heat transfer correlation because of its practical simplicity and proven applicability. Based on this correlation, the single-phase heat transfer coefficients in Eq. (4) $h_L$ and $h_G$ can be modeled as functions of Reynolds number, Prandtl number, and the ratio of bulk-to-wall viscosities. Thus, Eq. (4) can be expressed as

$$h_{TP} = (1-\alpha)h_L\left[1 + \frac{\alpha}{1-\alpha}\frac{\text{fctn}(Re, Pr, \mu_B/\mu_W)_G}{\text{fctn}(Re, Pr, \mu_B/\mu_W)_L}\right] \tag{5}$$

$$h_{TP} = (1-\alpha)h_L \times \left(1 + \frac{\alpha}{1-\alpha}\text{fctn}\left\{\left(\frac{Re_G}{Re_L}\right), \left(\frac{Pr_G}{Pr_L}\right), \left[\frac{(\mu_B/\mu_W)_B}{(\mu_B/\mu_W)_L}\right]\right\}\right) \tag{6}$$

Substituting the definition of Reynolds number ($Re = \rho V D/\mu_B$) for the gas ($Re_G$) and liquid ($Re_L$) yields

$$\frac{h_{TP}}{(1-\alpha)h_L} = \left(1 + \frac{\alpha}{1-\alpha} \times \text{fctn}\left\{\left[\frac{(\rho V D)_G}{(\rho V D)_L}\frac{(\mu_B)_L}{(\mu_B)_G}\right], \left(\frac{Pr_G}{Pr_L}\right), \left[\frac{(\mu_B/\mu_W)_G}{(\mu_B/\mu_w)_L}\right]\right\}\right) \tag{7}$$

Rearranging yields

$$\frac{h_{\mathrm{TP}}}{(1-\alpha)h_L} = \left(1 + \frac{\alpha}{1-\alpha} \times \mathrm{fctn}\left\{\left[\left(\frac{\rho_G}{\rho_L}\right)\left(\frac{V_G}{V_L}\right)\left(\frac{D_G}{D_L}\right)\right], \left(\frac{Pr_G}{Pr_L}\right), \left[\frac{(\mu_W)_L}{(\mu_W)_G}\right]\right\}\right) \tag{8}$$

where the assumption has been made that the bulk viscosity ratio in the Reynolds number term of Eq. (7) is exactly canceled by the last term in Eq. (7), which includes bulk viscosity ratio. Substituting Eq. (1) for the ratio of gas-to-liquid diameters ($D_G/D_L$) in Eq. (8) and based on practical considerations assuming that the ratio of liquid-to-gas viscosities evaluated at the wall temperature $[(\mu_W)_L/(\mu_W)_G]$ is comparable to the ratio of those viscosities evaluated at the bulk temperature ($\mu_L/\mu_G$), Eq. (8) reduces to

$$\frac{h_{\mathrm{TP}}}{(1-\alpha)h_L} = \left(1 + \frac{\alpha}{1-\alpha} \times \mathrm{fctn}\left\{\left[\left(\frac{\rho_G}{\rho_L}\right)\left(\frac{V_G}{V_L}\right)\left(\frac{\sqrt{\alpha}}{\sqrt{1-\alpha}}\right)\right], \left(\frac{Pr_G}{Pr_L}\right), \left(\frac{\mu_L}{\mu_G}\right)\right\}\right) \tag{9}$$

For use in further simplifying Eq. (9), combine Eqs. (2) and (3) for $V_G$ (gas velocity) and $V_L$ (liquid velocity) to get the ratio of $V_G/V_L$ and substitute into Eq. (9) to get

$$h_{\mathrm{TP}} = (1-\alpha)h_L \times \left\{1 + \mathrm{fctn}\left[\left(\frac{x}{1-x}\right), \left(\frac{\alpha}{1-\alpha}\right), \left(\frac{Pr_G}{Pr_L}\right), \left(\frac{\mu_L}{\mu_G}\right)\right]\right\} \tag{10}$$

Assuming that two-phase heat transfer coefficient can be expressed using a power-law relationship on the individual parameters that appear in Eq. (10), then Eq. (10) can be expressed as

$$h_{\mathrm{TP}} = (1-\alpha)h_L\left[1 + C\left(\frac{x}{1-x}\right)^m \left(\frac{\alpha}{1-\alpha}\right)^n \left(\frac{Pr_G}{Pr_L}\right)^p \left(\frac{\mu_L}{\mu_G}\right)^q\right] \tag{11}$$

where $h_L$ comes from the Sieder and Tate[12] equation as mentioned earlier. For the Reynolds number needed in that single-phase correlation, the following relationship is used to evaluate the in situ Reynolds number (liquid phase) rather than the superficial Reynolds number ($Re_{\mathrm{SL}}$) as commonly used in the correlations of the available literature (see Ref. 1):

$$Re_L = \left(\frac{\rho V D}{\mu_B}\right)_L = \frac{4\dot{m}_L}{\pi\sqrt{1-\alpha}(\mu_B)_L D} \tag{12}$$

Any other well-known single-phase turbulent heat transfer correlation could have been used in place of the Sieder and Tate[12] correlation. The difference resulting from the use of a different single-phase heat transfer correlation will be absorbed during the determination of the values of the leading coefficient and exponents on the different parameters in Eq. (11).

To determine the values of leading coefficient and the exponents in Eq. (11), four sets of experimental data (a total of 255 data points) from three available experimental studies[3–5] were used in this study. The reported experimental uncertainties of the two-phase heat transfer coefficients from those studies are ±4–16% for Aggour,[3] ±4.5–14% for Vijay,[4] and ±6.9–21.1% for Rezkallah,[5] respectively. The ranges of these four sets of experimental data can be found in Kim et al.[1] The experimental data included four different liquid-gas combinations (water–air, silicone–air, water–helium, water–freon 12) and covered a wide range of variables, including liquid and gas flow rates, properties, and flow patterns. The selected experimental data were only for turbulent two-phase heat transfer data in which the superficial Reynolds numbers of the liquid ($Re_{SL}$) were all greater than 4000.

## Prediction Results and Discussion

A single correlation that can be used to predict turbulent two-phase gas–liquid nonboiling heat transfer rate for wide range of fluid combinations and several different flow patterns was obtained by curve-fitting Eq. (11) to the 255 data points obtained from the literature using the least-squares method. The best-fit correlation was found to be

$$\frac{h_{\mathrm{TP}}}{(1-\alpha)h_L} = \left[1 + 0.27\left(\frac{x}{1-x}\right)^{-0.04}\left(\frac{\alpha}{1-\alpha}\right)^{1.21} \times \left(\frac{Pr_G}{Pr_L}\right)^{0.66}\left(\frac{\mu_G}{\mu_L}\right)^{-0.72}\right] \tag{13}$$

where the parametric ranges were

$$4000 < Re_{\mathrm{SL}} < 1.26 \times 10^5, \qquad 8.4 \times 10^{-6} < x/(1-x) < 0.77$$

$$0.01 < \alpha/(1-\alpha) < 18.61, \qquad 1.18 \times 10^{-3} < Pr_G/Pr_L < 0.14$$

$$3.64 \times 10^{-3} < \dot{m}_G/\dot{m}_L < 0.02$$

In the development of Eq. (13), the values of the void fraction ($\alpha$) were directly taken from the original experimental data.[3–5] These $\alpha$ values were calculated by the original investigators based on the equation provided by Chisholm,[13] which can be expressed as

$$\alpha = \{1 + (V_G/V_L)[(1-x)/x](\rho_G/\rho_L)\}^{-1} \tag{14}$$

**Table 2 Results of the predictions for available two-phase heat transfer experimental data, using the recommended new correlation [Eq. (13)]**

| Fluids ($Re_{\mathrm{SL}} > 4000$) | Mean dev., % | rms dev., % | Number of data within ±30% | Range of dev., % |
|---|---|---|---|---|
| All of the data points in Table 1 255 data points | 2.54 | 12.78 | 245 | −64.71 and 39.55 |
| Water–air 105 data points Vijay[4] | 3.53 | 12.98 | 98 | −34.97 and 39.55 |
| Silicone–air 56 data points Rezkallah[5] | 5.25 | 7.77 | 56 | −7.25 and 12.13 |
| Water–helium 50 data points Aggour[3] | −1.66 | 15.68 | 48 | −64.71 and 32.19 |
| Water–freon 12 44 data points Aggour[3] | 1.51 | 13.74 | 43 | −24.51 and 32.96 |

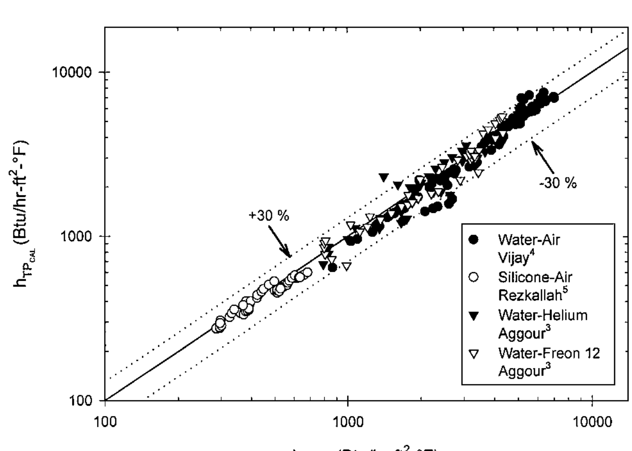


**Fig. 1 Comparison of the new correlation [Eq. (13)] with all of the two-phase heat transfer experimental data (255 data points) in Table 1.**

**Table 3 Summary of the values of the leading coefficient *C* and exponents *m*, *n*, *p*, *q* in the new heat transfer coefficient correlation $h_{TP}$ [Eq. (11)] and the prediction results**

| Fluids ($Re_{SL} > 4000$) | Value of *C* and exponents (*m*, *n*, *p*, *q*) *C* | *m* | *n* | *p* | *q* | Mean dev., % | rms dev., % | Number of data within ±30% | Range of dev., % |
|---|---|---|---|---|---|---|---|---|---|
| All of the data points in Table 1 255 data points | 0.27 | −0.04 | 1.21 | 0.66 | −0.72 | 2.54 | 12.78 | 245 | −64.71 and 39.55 |
| Water–air 105 data points Vijay[4] | 16.69 | −0.32 | 1.65 | 1.23 | 0.40 | 3.22 | 8.04 | 105 | −18.25 and 27.0 |
| Silicone–air 56 data points Rezkallah[5] | 2.19 | 0.40 | 0.21 | 0.87 | −0.96 | 0.55 | 3.38 | 56 | −5.37 and 10.34 |
| Water–helium 50 data points Aggour[3] | 61.16 | −0.29 | 1.58 | 0.24 | 1.47 | 3.03 | 12.24 | 49 | −28.05 and 34.92 |
| Water–freon 12 44 data points Aggour[3] | 599.9 | −0.30 | 1.64 | 5.27 | −0.85 | 1.67 | 11.56 | 44 | −25.04 and 28.42 |

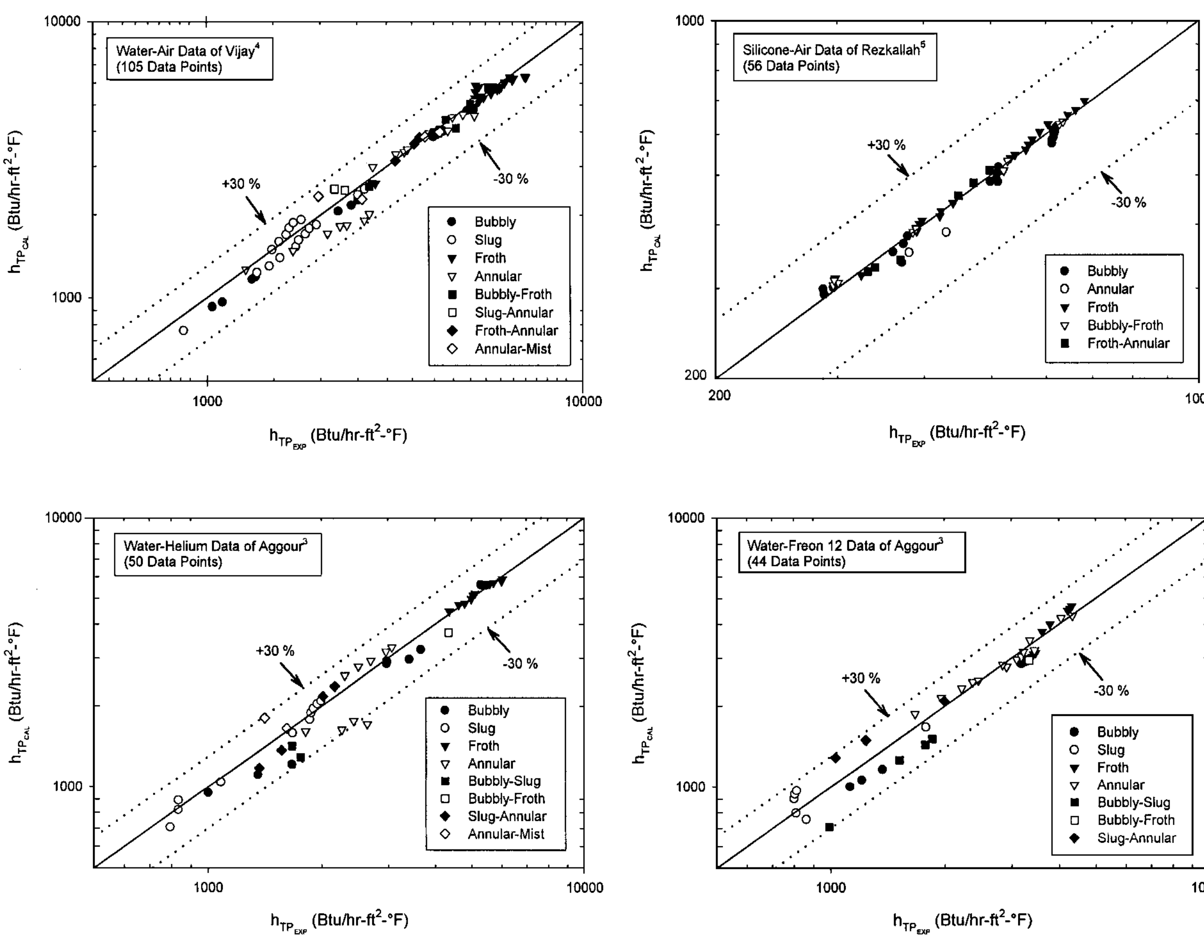


**Fig. 2 Comparison of the fluid-dependent new correlations for each of the four different fluid combinations with the available experimental data.**

Comparing Eq. (13)'s predicted heat transfer coefficients to the experimentally determined values (see Tables 2 and 3) yields a mean deviation of 2.54%, an rms deviation of 12.78%, and a deviation range of −64.71–39.55%. The exponent value on the parameter $[x/(1-x)]$ in Eq. (13) has a very small magnitude (0.04) for the sets of experimental data used in this study. However, even with a small exponent, this term still appears to play a very important role because complete elimination of the parameter yielded substantial underpredictions, resulting in the best achievable mean deviation being 10.55% with a corresponding rms deviation of 15.6%. This caused about 89% of the experimental data (224 data points) to be under predicted. Figure 1 shows how well the general correlation predicted the four different sets of gas–liquid experimental data. About 83% of the data (212 data points) were predicted with less than ±15% deviation, and about 96% of the data (245 data points) were predicted with less than ±30% deviation.

Table 2 also shows the results of the general correlation's predictions of the individual data sets. About 93% of the water–air data of Vijay[4] (98 data points out of 105), 100% of the silicone–air data of Rezkallah[5] (56 data points), 96% of the water–helium data of Aggour[3] (48 data points out of 50), and 98% of the water–freon 12 data of Aggour[3] (43 data points out of 44) were predicted with less than ±30% deviation. Comparing the performance of Eq. (13) with the correlations listed in Table 1 (Refs. 3, 6–8) for the specific fluid combinations given clearly shows that the new correlation is as good or better than the specific fluid-dependent correlations in predicting the experimental data. Specifically, for the water–air experimental data of Vijay[4] and the silicone–air experimental data of Rezkallah,[5] the performance of the new correlation is significantly better than the fluid combination specific correlations of Shah[6] for water–air and Rezkallah and Sims[7] for silicone–air that were recommended in Table 1.

To further improve the capabilities of the general correlation in predicting the specific fluid combination experimental data, the general form of the correlation [Eq. (11)] was curve-fitted to each of the four different fluid combination experimental data sets. Table 3 gives a summary of the resulting curve-fitted values of the leading coefficient $C$ and the exponents $m$, $n$, $p$, and $q$, and the prediction results for each of the four fluid combinations in terms of mean deviation, rms deviation, range of deviation, and the number of data points within ±30% deviation. For comparison purposes the table also lists the values of the curve-fitted parameters for the improved correlation [Eq. (13)] and the prediction results. Figure 2 provides a comparison of the predictions of the fluid-dependent improved correlations with the four corresponding fluid combinations' experimental data from the literature.

As can be seen from Table 3, for the water–air data of Vijay[4] the correlation predicts the heat transfer coefficient of the 105 experimental water–air data points with better mean and rms deviations, while providing a more balanced deviation range than the previously recommended correlation of Shah[6] in Table 1. About 94% of the data (99 data points out of 105) were predicted with less than ±15% deviation. For the silicone–air experimental data of Rezkallah,[5] the correlation prediction results for the 56 experimental silicone–air data points are much improved in all respects as compared to the results of the previously recommended correlation of Rezkallah and Sims[7] in Table 1. All of the data (56 data points) were predicted with less than ±15% deviation.

For the water–helium experimental data of Aggour,[3] the correlation predicts the 50 data points with the measures of accuracy showing significant improvement, compared with these same measures from the Knott et al.[8] correlation of Table 1. About 86% of the data (43 data points out of 50) was predicted with less than ±15% deviation. For the water–freon 12 experimental data of Aggour,[3] the correlation prediction results for the 44 experimental water–freon 12 data were comparable to those of the previously recommended correlation of Aggour[3] in Table 1. About 86% of the data (36 data points out of 44) were predicted with less than ±15% deviation.

The tabulated curve-fitted values, for the fluid combination dependent improved correlations summarized in Table 3, provide an indication of the effects of changing the gas–liquid combination. As shown, these effects can vary widely depending on the properties of the gas and liquid. The variation in the exponent values $m$ and $n$ for the parameters $[x/(1-x)]$ and $[\alpha/(1-\alpha)]$ was much smaller than the variation in the exponents of the other parameters in the correlation. From this, the conclusion can be made that the effects of these parameters on the two-phase heat transfer have a weaker dependence on the gas properties than the other parameters in the general correlation. Also, the observation can be made from Table 3 that the magnitude of the leading coefficient $C$ is considerably larger when helium or freon 12 is mixed with water in two-phase flow. Based on this observation, it appears that those parameters in the new correlation, representing mixing effects of gas liquid on the two-phase heat transfer, can contribute more to the correlation if an inert or relatively inert type of gas is mixed with the liquid.

## Conclusions

We have developed a much improved robust heat transfer correlation [see Eq. (13)], which can be applied to turbulent gas–liquid two-phase flow in vertical pipes with different fluid flow patterns and fluid combinations. The new correlation predicts very well the heat transfer coefficient for the 255 experimental data points referred to in Table 1 for water–air, silicone–air, water–helium, and water–freon 12, having an overall mean deviation of 2.54%, an rms deviation of 12.78%, and a deviation range of −64.71–39.55%. Additional improvements in the predictive capability of the improved correlation can be obtained by using the fluid-dependent curve-fitting coefficients listed in Table 3. The conclusion was made from the observations of the variations in the exponent values on the parameters in the fluid combination dependent improved correlations that the effects of the parameters $[x/(1-x)]$ and $[\alpha/(1-\alpha)]$ on two-phase heat transfer had a weaker dependence upon the gas properties than the other parameters in the improved correlation. Also, the observation was made that those parameters in the new overall correlation, representing mixing effects of gas–liquid on the two-phase heat transfer, can contribute more to the correlation if an inert or relatively inert type of gas (helium and freon 12) is mixed with the liquid (water).

## Acknowledgments

The authors gratefully acknowledge the financial assistance provided by the members of the University of Tulsa Joint Industry Project on Multiphase Heat Transfer in Flowlines and Wellbores.

## References

[1]Kim, D., Ghajar, A. J., Dougherty, R. L., and Ryali, V. K., "Comparison of Twenty Two-Phase Heat Transfer Correlations with Seven Sets of Experimental Data, Including Flow Pattern and Tube Orientation Effects," *Heat Transfer Engineering*, Vol. 20, No. 1, 1999, pp. 15–40.

[2]Kim, D., Ghajar, A. J., and Dougherty, R. L., "Development of Improved Two-Phase Two-Component Pipe Flow Heat Transfer Correlations from Existing Correlations and Published Data," *Proceedings of the Fifth ASME/JSME Joint Thermal Engineering Conference*, AJTE-99-6122, American Society of Mechanical Engineers, New York, 1999.

[3]Aggour, M. A., "Hydrodynamics and Heat Transfer in Two-Phase Two-Component Flow," Ph.D. Dissertation, Mechanical Engineering Dept., Univ. of Manitoba, Winnipeg, MB, Canada, June 1978.

[4]Vijay, M. M., "A Study of Heat Transfer in Two-Phase Two-Component Flow in a Vertical Tube," Ph.D. Dissertation, Mechanical Engineering Dept., Univ. of Manitoba, Winnipeg, MB, Canada, Dec. 1978.

[5]Rezkallah, K. S., "Heat Transfer and Hydrodynamics in Two-Phase Two-Component Flow in a Vertical Tube," Ph.D. Dissertation, Mechanical Engineering Dept., Univ. of Manitoba, Winnipeg, MB, Canada, Dec. 1986.

[6]Shah, M. M., "Generalized Prediction of Heat Transfer During Two Component Gas-Liquid Flow in Tubes and Other Channels," *AIChE Symposium Series*, Vol. 77, No. 208, 1981, pp. 140–151.

[7]Rezkallah, K. S., and Sims, G. E., "An Examination of Correlations of Mean Heat Transfer Coefficients in Two-Phase and Two-Component Flow in Vertical Tubes," *AIChE Symposium Series*, Vol. 83, No. 257, 1987, pp. 109–114.

[8]Knott, R. F., Anderson, R. N., Acrivos, A., and Petersen, E. E., "An Experimental Study of Heat Transfer to Nitrogen-Oil Mixtures," *Industrial and Engineering Chemistry*, Vol. 51, No. 11, 1959, pp. 1369–1372.

[9]Govier, G. W., and Aziz, K., '*The Flow of Complex Mixtures in Pipes*, Reinhold, New York, 1973.

[10]Griffith, P., and Wallis, G. B., "Two-Phase Slug Flow," *Journal of Heat Transfer*, Vol. 83, *Series C*, 1961, pp. 307–320.

[11]Hewitt, G. F., and Hall-Taylor, N. S., *Annular Two-Phase Flow*, Pergamon, Oxford, 1970.

[12]Sieder, E. N., and Tate, G. E., "Heat Transfer and Pressure Drop of Liquids in Tubes," *Industrial and Engineering Chemistry*, Vol. 28, No. 12, 1936, pp. 1429–1435.

[13]Chisholm, D., "Research Note: Void Fraction During Two-Phase Flow," *Journal of Mechanical Engineering Science*, Vol. 15, No. 3, 1973, pp. 235, 236.